How to estimate tidal amplitude

According to the law of universal gravitation discovered by Newton, every body exerts a force of attraction proportional to its mass and decreasing with distance r in $1/r^2$. Newton also showed that the attraction due to all parts of the Earth, supposedly homogeneous, is equal to what it would be if the whole mass were concentrated in its centre, i.e. at R = 6370 km from the surface. The mass of the Moon is about m/M = 1.2% of that of the Earth, and located at D = 380,000 km, about 60 times more than the distance R to the centre of the Earth (the diagram is not to scale). Thus the force of gravity (per unit mass) exerted by the Moon is $(m/M)(R/D)^2$ g = 3 x 10^{-6} g.

A hasty reasoning would suggest that this force lifts the ocean on the moon side and digs it on the opposite side. However, the main effect of the lunar attraction is to rotate the Earth around the barycentre G of the Earth-Moon system symmetrically with the rotation of the Moon around this same centre (located at a distance Dm/M = 4600 km from the centre of the Earth). The residual lunar gravity that we feel is therefore the difference between the gravity depending on the local distance to the Moon D-x (see figure), in $1/(D-x)^2$, and the gravity controlling the global movement of the Earth, applied in its center at distance D from the Moon, therefore in $1/D^2$. With the approximation $1/(D-x)^2$-1-1/D2 ≃ $2x/D^3$, the effective force per unit of mass that lifts us to the Moon is therefore reduced, for x = R, to $2(m/M)(R/D)^3$ g=10^{-7} g. On the opposite side, x = -R, the lack of lunar attraction leads to a reduction of the same value, hence a bulge on each side.

The effect, equivalent to a mass variation of 10 mg for a 100 kg man, is hardly noticeable (but quite measurable by gravimeters). A **potential energy** $-(m/M)(R^2/D^3)(x^2/R)$ g can be associated with the tidal force, which is minimum at x = +-R, corresponding to an elevation $(m/M)R(R/D)^3$ = 0.35 m (at the equator).

For the Sun, with mass m/M = 330,000 relative to the Earth, at a distance D = 150,10^6 km, we have $(m/M)(R/D)^2$ g = 10^{-4} g, hence a force 30 times greater than that of the Moon. But we must again remove the average effect on the Earth (we are in orbit around the Sun like the Earth), and the elevation becomes $(m/M)R(R(R/D)^3$ = 0.16 m, a factor 2.2 lower than the lunar effect. Depending on whether the Sun's effect is added to or subtracted from that of the Moon, the predicted tidal range varies by a factor of 2.5, from 20 cm to 50 cm at the equator. This corresponds indeed to typical values observed in the deep ocean. Near the coast the amplitude is generally amplified as the tidal wave propagation piles up on shallower waters.

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